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A Comparison of Meso-Scaled Heat Flux and Temperature in Fully-Developed Turbulent Pipe and Channel Flows

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Abstract

Data from direct numerical simulations (DNS) of constant surface heat-flux in fully-developed turbulent pipe and channel flows is used to explore the physical mechanisms of turbulent heat transfer. The analysis employs a theory based on the magnitude ordering of terms in the mean thermal energy equation of wall-bounded turbulent heat transfer, Wei *et al.* [10]. Identifying the leading order terms in the mean energy equation reveals a four layer structure similar to that found for the mean momentum equation. The traditional inner scale is then transformed into new inner length and then the invariant form admitted by the relevant form of the mean energy equation is determined. These apply to inner, outer and intermediate regions of the flow, whose properties are dependent on a small parameter that is a function of Peclet number. Existing and new DNS data of turbulent heat transfer for both channel and pipe flow are shown to support the scalings derived from the theory. The analysis reveals that the balance breaking and exchange of terms in the mean energy equation that occurs across the intermediate mesolayer is similar to that in the mean momentum equation.

Introduction

Wall-bounded turbulent flows are present in a large number of industrial and technological applications which involve heat and mass transport. Thus, it is important to understand the proper scaling of the thermal transport in order to accurately represent the effect of the governing parameters on the thermal field statistics. In this regard, recent analyses of the mean momentum equation have been used to explore the underlying physics and scaling of turbulent wall-flows. Wei *et al.* [11] introduced a generic first-principles framework, an extension of which leads to a mesoscaling of Reynolds shear stress (Wei *et al.* [12]) and mean velocity field (Wei *et al.* [13]) in turbulent channel and pipe flows. Existing DNS data were shown to be consistent with this mesoscaling over a spatial domain extending from at least the lower boundary of the mesolayer (layer III herein) to the centerline.

Traditional representations of temperature and turbulent heat flux profiles generally employs either inner or outer normalizations. These normalizations, however, fail to provide invariant profiles as the relevant non-dimensional parameters are varied. Under inner normalization, the mean temperature is normalised by the so-called friction temperature (defined later) and the wall distance is normalised by the friction velocity u_τ and the kinematic viscosity ν. This normalization, however, is only relevant over a small region near the wall that encompasses the conductive sublayer ($y^+ \approx 5$)[3]. Moreover, the logarithmic layer data exhibit different mean temperature profiles as a function of both Reynolds and Prandtl numbers. The existence of this richer range of phenomena from the momentum case essentially arises from the extra parameter, Peclet number (product of Reynolds and Prandtl number) in the mean energy equation.

The mean energy balance equation is governed by the balance between the molecular diffusion, turbulent transport and mean streamwise advection. According to the present theory, the transition from inner to outer scaling physically takes place owing a balance breaking and exchange of these mechanisms as a function of scale. This underlies the existence of an intermediate region between inner and outer layers (thermal mesolayer) where, in the mean, all these three terms are nearly in balance, Wei *et al.* [10]. Similarly, by assuming the existence of overlap layers Seena *et al.* [9] construct a closure model that leads to profiles for mean temperature and turbulent heat fluxes. The present framework only relies on the magnitude ordering of the terms in the mean energy equation, and thus does not invoke additional assumption or resort to the use of a closure model.

Numerical Procedures and Data Sets

The numerical scheme used for obtaining DNS data of turbulent flow and heat transfer in a pipe is detailed in Saha *et al.*[8]. The numerical algorithm is based on a cylindrical coordinate spectral element/Fourier spatial discretisation [1]. A good number of checks have been carried out to ensure enough resolution and the validity of the present dataset. The onset of the four layer structure for hydrodynamic flow fields of both pipe and channel occurs at about $Re_\tau = 180$ (Klewicki *et al.* [6] and Elsnab *et al.* [2]) and hence, the selection of the present data sets ensures the existence of a nascent four layer regime for the momentum field. Table 1 shows the present list of DNS datasets for both pipe and channel flow. High resolution DNS data of turbulent heat transfer in channel is extracted from Kawamura's group (Kawamura *et al.* [4, 5] and Kozuka *et al.* [7]).

Re_τ	Pr	Pe_τ	*Channel*	*Pipe*
180	0.025	4.5	□	∀
180	0.05	9.0	▼	★
180	0.1	18	◀	▲
180	0.2	36	∘	∅
180	0.4	72	⊗	◇
180	0.6	108	Θ	■
180	0.71	127.8	⊕	Ψ
180	1.0	180	χ	◁
180	2.0	360	♣	$
180	5.0	900	●	▷
180	7.0	1260	♦	♠
395	0.025	9.875	▶	©
395	0.71	280.45	®	▽

Table 1: DNS database for Turbulent Heat Transfer in Pipe and Channel flow. The symbols for channel and pipe flow of each condition are used as legends for the subsequent figures.

Mean Thermal Energy Balance Framework

The conventional form of outer normalised Reynolds averaged energy balance equation is found by using the pipe radius or channel half height δ to normalise the wall distance $\eta = y/\delta$. This gives

$$\sigma^2 \frac{d^2\Theta^+}{d\eta^2} + \frac{d\Upsilon^+}{d\eta} + R(\eta) = 0, \tag{1}$$

where Θ^+ is the non-dimensional mean temperature normalized by the friction temperature $\Theta_\tau = q_w/\rho C_p u_\tau$, q_w is the heat flux applied on the pipe or channel outer walls, ρ is the mass density, C_p is the specific heat, u_τ is the friction velocity, $R(\eta) = \frac{2}{U_b}\left[\frac{1}{(\eta-1)^2}\int_1^{1-\eta}(\eta-1)\,U d\eta - U\right]$ for pipe flow and $R(\eta) = U(\eta)/U_b$ for channel flow, U_b is the bulk mean velocity, $\Upsilon^+ = \langle -v^+\theta^+\rangle$ is the turbulent radial heat flux and the small parameter σ is defined by

$$\sigma = \sqrt{\frac{1}{Pe_\tau}} = \sqrt{\frac{1}{Pr\delta^+}}, \tag{2}$$

where Pr is the Prandtl number and the wall Reynolds number is Re_τ or $\delta^+ = u_\tau\delta/\nu$, also called the inner normalised pipe radius or channel half height. Equation (1) implies a fully developed thermal field hence there is no dependence on axial direction. This equation is valid for both pipe and channel flows. At sufficiently high Pe_τ, $R(\eta)$ is $O(1)$ for all values of η except in the region interior to the peak in the turbulent heat flux profile. Equation (1) also expresses a balance between mean streamwise advection and turbulent transport flux gradient for sufficiently small values of σ^2.

The conventional inner scaled mean energy equation for fully developed wall bounded turbulent heat transfer is

$$\frac{1}{\mathrm{Pr}}\frac{d^2\Theta^+}{dy^{+2}} + \frac{d\Upsilon^+}{dy^+} + R\left(y^+\right) = 0, \tag{3}$$

where $y^+ = yu_\tau/\mu$ is the inner-normalised wall-normal distance and μ the kinematic viscosity.

Based upon the mean momentum equation analysis of Wei *et al.* [11], we propose an alternative form of the inner normalised energy equation. This form employs a new inner variable parameter, $y_\sigma = \eta/\sigma^2$, which follows from the work of Wei *et al.* [10]. This yields a new "inner" form for the mean energy balance equation:

$$\frac{d^2\Theta^+}{dy_\sigma{}^2} + \frac{d\Upsilon^+}{dy_\sigma} + \sigma^2 R_\sigma(y_\sigma) = 0, \tag{4}$$

with boundary conditions, $\Theta^+ = 0$, $d\Theta^+/dy_\sigma = 1$ at $y_\sigma = 0$. Physically, y_σ reflects the scale separation associated with increasing Peclet number. For large Peclet number, equation (4) indicates a balance between the molecular diffusion flux gradient and the turbulent transport flux gradient.

Traditional Scaling Analysis of Heat Transfer

The conventional way of presenting turbulence statistical profiles uses a combination of inner- and outer-normalisations. Figures 1 and 2 show the inner and outer-normalised turbulent radial heat flux profiles. Existing (channel) and present (pipe) data reveal that the traditional inner normalisation fails to yield an invariant profiles for varying Reynolds and Prandtl numbers over more a couple of y^+ units from the wall. Traditional outer normalization, however, yields a might tighter clustering of the turbulent radial heat flux profiles over an outer domain that extends from the centerline inward for $Pe_\tau \geq 72$. Neither of these normalizations yields an invariant profile in the vicinity of the peak radial heat flux. It is also apparent from figure 1 that the

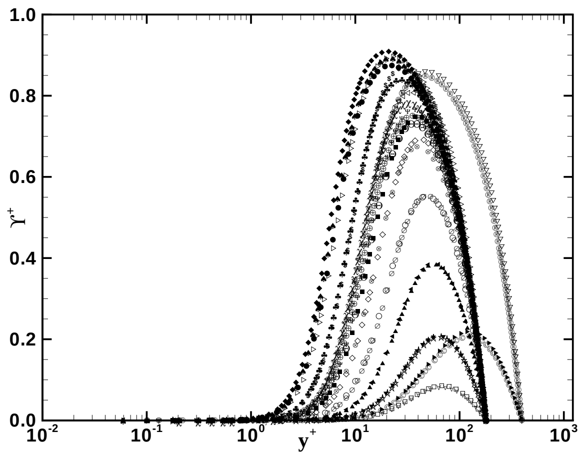

Figure 1: Traditional inner scaling of turbulent heat flux.

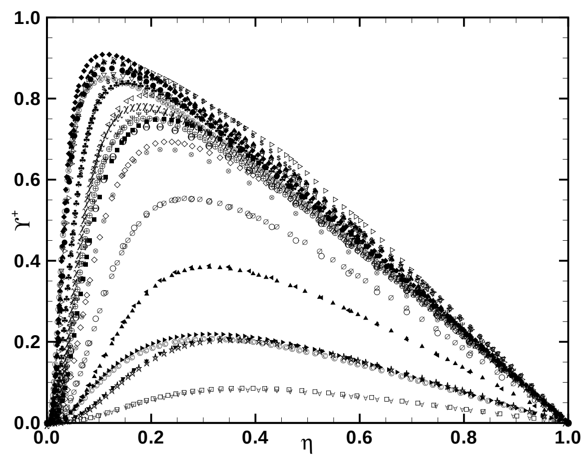

Figure 2: Traditional outer scaling of turbulent heat flux.

inner normalised maximum turbulent radial heat flux location moves outward with increasing Peclet number, while the outer normalized peak location moves inward (figure 2).

At large Peclet number, when the value of σ^2 becomes very small and $R(\eta)$ is $O(1)$, the $O(\sigma^2)$ term in the outer normalised mean energy balance may be neglected. This leaves

$$\frac{d\Upsilon^+}{d\eta} + R(\eta) = 0. \tag{5}$$

Integrating and using the boundary condition yields

$$\Upsilon(\eta) = -\int_1^\eta R(\eta)d\eta = 1 - \eta, \tag{6}$$

indicating a linear variation of turbulent radial heat flux independent of Peclet number. This condition is satisfies in the domain where, according to equation (5), the mean streamwise advection and turbulent transport flux gradient are nominally in balance. It is important to note that the peak values of the profiles of figure 2 increases and apparently approaches unity with increasing Peclet number. It is also apparent that at comparable parameter values the pipe and channel flow profiles are convincingly the same.

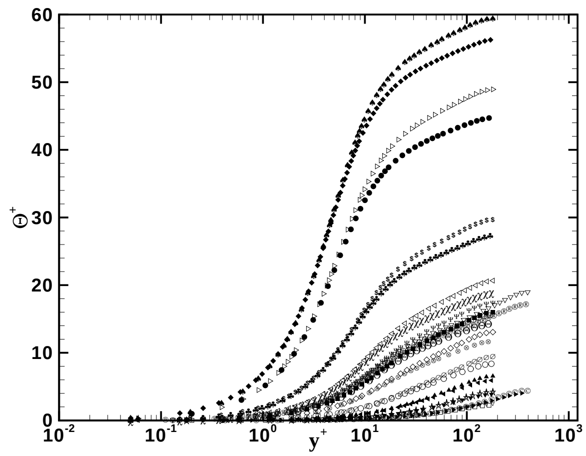

Figure 3: Traditional inner scaling of mean temperature.

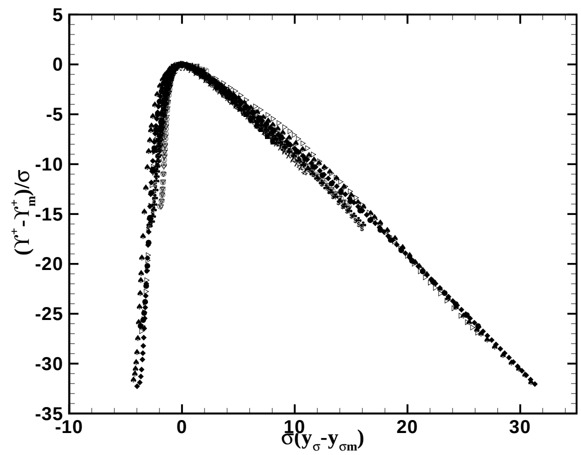

Figure 5: Mesoscaling of turbulent heat flux.

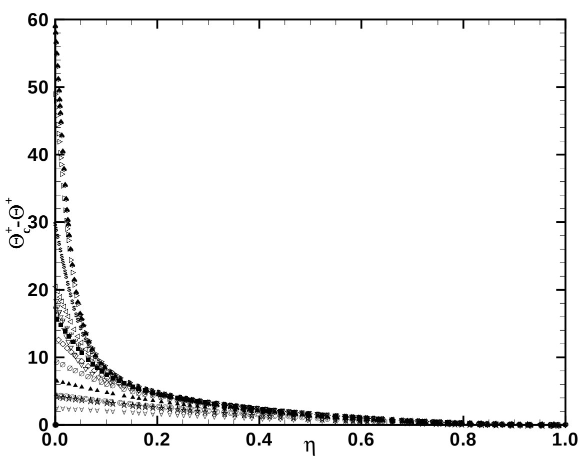

Figure 4: Traditional outer scaling of mean temperature.

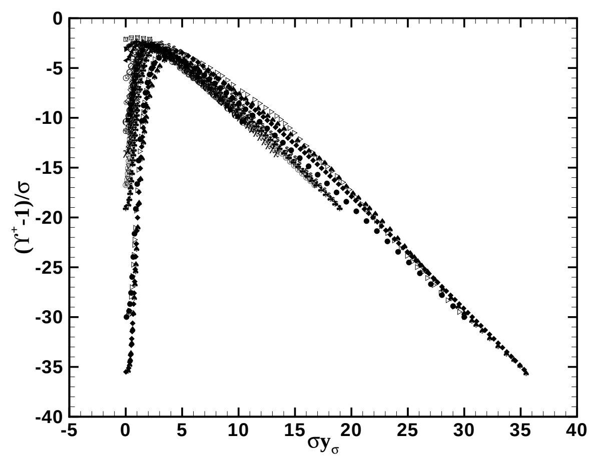

Figure 6: Approximate mesoscaling of turbulent heat flux.

Like the turbulent heat flux profiles, the mean temperature profiles fail to coincide under inner-normalisation, except very near the wall (figure 3). Again, this may be traced back to their dependence on both Reynolds and Prandtl numbers. Traditionally, four different thermal wall layers are identified by Kader [3]: the molecular transport sublayer, the buffer layer, the logarithmic layer and the outer layer. Apart from a wall-normal narrow range within the molecular sublayer, the mean temperature profiles do not show any well-defined trends. One of the reasons is that the normalisation parameter does not include the effect of Prandtl number explicitly. Interestingly, there exists a distinct difference between the pipe and channel flow temperature profiles from the logarithmic layer at higher Peclet numbers. On the other hand, the data of figure 4 suggest an invariant outer-normalized temperature profile for $\eta > 0.5$.

Mesoscaling Analysis of Heat Transfer

The new inner scaled mean energy balance (equation (4)) at large Peclet number (small σ^2) indicates that the leading order balance is between molecular diffusion and mean advection. Near the location of the peak turbulent heat flux, however, all three of the terms in equation (4) have the same order of magnitude. This is similar to the property exhibited by the mean momentum equation (see, Wei *et al.*[11]), indicating that a four layer structure is also applicable to the mean energy equation.

The goal now is to rescale equation (4) such that all the terms are formally $O(1)$ in layer III. Following Wei *et al.*[11], a successful rescaling will take the form

$$\hat{y_\sigma} = \sigma(y_\sigma - y_{\sigma m}), \hat{\Upsilon} = (1/\sigma)(\Upsilon^+ - \Upsilon^+_m), \tag{7}$$

where $y_{\sigma m}$ and Υ^+_m are the peak turbulent radial heat flux location and value, respectively. Normalization of the mean energy equation according to these variables results in

$$\frac{d^2\Theta^+}{d\hat{y}_\sigma^2} + \frac{d\hat{\Upsilon}}{d\hat{y}_\sigma} + 1 = 0. \tag{8}$$

The desired invariant form is attained, as all of the normalized terms are formally $O(1)$. Note that similar to the mean momentum equation, it is not necessary to rescale the mean temperature to attain an invariant form that reflects the true magnitude ordering of terms. Note also that $d\Upsilon^+/d\eta$ is identically equal to $d\hat{\Upsilon}/d\hat{y}_\sigma$, and thus the meso equation (8) transparently matches the outer equation (1). This indicates that the mesoscaling should be appropriate for the turbulent radial heat flux data into the outer region as well. Figure 5 shows the mesoscaled turbulent radial heat flux profiles. All of the profiles collapse onto a single curve under the coordinate stretching produced by the meso-variables $\hat{y}_\sigma$ and $\hat{\Upsilon}$. This scaling for turbulent radial heat flux is apparently valid over an interior region the extends from inside the peak in Υ^+ to a zone near the centerline. In

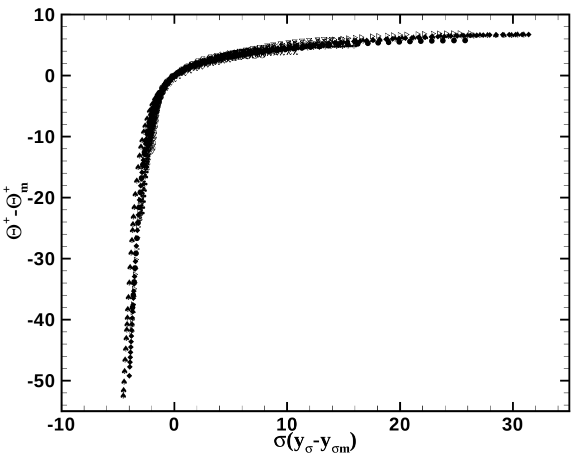

Figure 7: Mesoscaling of normalised mean temperature.

general, the theory indicates that this scaling should be valid in a domain surrounding $y_\sigma m(\hat{y}_\sigma = 0, \hat{\Upsilon} = 0)$ having an extent of $\Delta\hat{y}_\sigma = O(1)$. As noted previously, however, this scaling naturally melds with outer scaling, and thus in these coordinates it is analytically predicted to extend to the centerline. Within the mesoscaling domain, no significant differences between pipe and channel flow data are observed. For the present data sets, this domain extends from $10 < \hat{y}_\sigma < 32$. The indicate scaling does not hold in a narrow region near the wall. This is where the scaling patch affiliated with inner length is known to hold. This behaviour is also similar to the mesoscaling of Reynolds shear stress as explained by Wei *et al.*[12].

In order to evaluate the mesoscaled variables $\hat{y}_\sigma$ and $\hat{\Upsilon}$, one should require prior knowledge of the maximum turbulent radial heat flux value and its location. Due to the limitation of precisely determining the value of $y_{\sigma m}$ and Υ_m^+, Wei *et al.*[12] proposed an alternative approach to express the 'approximate' mesoscaling behaviour. This scaling is based upon the limiting behaviours of $y_{\sigma m}$ and Υ_m^+ which can be estimated for sufficiently high Peclet number as follows

$$y_{\sigma m} = O(1/\sigma), \Upsilon_m^+ = 1 - O(\sigma), \tag{9}$$

and equation (7) then yields

$$\hat{y_\sigma} = \sigma y_\sigma - O(1), \hat{\Upsilon} = (1/\sigma)(\Upsilon^+ - 1) + O(1). \tag{10}$$

Thus an approximate mesoscaling can be constructed by plotting $(\Upsilon^+ - 1)/\sigma$ versus σy_σ, without necessarily knowing the value of $y_{\sigma m}$ and Υ_m^+. As shown in figure 6, the turbulent heat flux profiles nominally merge to a single curve, particularly for the higher Peclet number. This approximate scaling is expected to improve with increasing Peclet number. Like the heat flux profile, the mesoscaled mean temperature profiles convincingly support the theory, as the profiles of figure 7 increasingly collapse onto a single curve with increasing Peclet number.

Conclusions

The mesoscaling analysis of turbulent heat transfer has been successfully validated using existing DNS data. The present methodology scaled the radial heat flux, and the mean temperature over a considerable domain centered about the peak heat flux location. The present framework is analogous to that used for the Reynolds shear stress by Wei *et al.*[12], and also reinforces their earlier findings for the turbulent heat flux, [11] . The present analyses also suggests that, at high Peclet number, the scaling characteristics of the temperature field become increasingly similar to those of the momentum field at high Reynolds number.

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